# RESOLUTION OF THE COSMOLOGICAL CONSTANT PROBLEM

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Abstract: We examine the cosmological constant using the standard approach that is currently Considered to be the conventional wisdom to achieve a theoretical prediction for it. The standard approach has shortcomings which lead to what is called 'The worst catastrophe in theoretical physics' in that the ratio of the theoretical prediction $\rho_{\Lambda}^{theory}$ to the observed value $\rho_{\Lambda}^{obs}$ is $\rho_{\Lambda}^{theory}/\rho_{\Lambda}^{obs} \sim 10^{120}$ . We examine the vacuum fluctuations of the mass spectra and of the string modes in the RNS Sector to bring about exact agreement between $\rho_{\Lambda}^{theory}$ and $\rho_{\Lambda}^{obs}$ with the added benefit that this agreement fixes $m_{v_e}$ the mass of the electron's neutrino.

PHYSH-General Relativity, Cosmology, Particle Physics in the early universe.

## 1. *Introduction*

a) The Einstein Field Equations

Einstein's original field equations are [1]:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G T_{\mu\nu} \quad . \tag{1}$$

Where in equation (1) and in the equations to follow we adopt the convention ħ = c =1 Unless stated otherwise for clarity.However we will keep Newton's constant.

Making the assumption that since we will be considering very large scales where the universe is



**NOTE: See errata at end of paper.**

homogeneous and isotopic we may postulate the Robertson-Walker metric as being applicable:

$$ds^2 = -dt^2 + a^2(t)R_0^2[dr^2/1-kr^2 \quad + r^2 d\Omega^2] \qquad (2)$$

Where $d\Omega^2 = d\theta^2 + sin^2\theta \ d\emptyset^2$ is the metric on a two-sphere. K is the curvature parameter which Takes the values +1 , 0 or -1 for positively curved, flat or negatively curved space respectively. a(t)= R(t)/$R_0$. z is the redshift factor and is related to the scale factor fofr the time at which it was emitted by:

$$a = \frac{1}{1+z} \qquad . \qquad (3)$$

Note: the energy-momentum sources are usually assumed to be a perfect fluid of energy-density P and isotropic pressure p and it can be shown that

$$T_{\mu\nu} = (p+\rho)\ U_\mu U_\nu + p g_{\mu\nu} \qquad (4)$$

With $U_\mu$ being the fluid four-velocity.

b) Introducing The Friedmann Equations

For the case in which the rest frame of the fluid is that of a co-moving observer In equation (2) one obtains a Robertson-Walker solution to Einstein's equations with them redu-Cing to the two Friedmann equations:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2 R_0^2} \qquad (5)$$

Where H is the Hubble parameter and after some manipulation one finds that

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad . \qquad (6)$$

c) The need for a static universe. Introduction of the Cosmological Constant

In hopes that General Relativity would be consistant with Mach's Principle, as well as the time's astronomical data , Einstein sought static solutions of equations (5) and (6) $\dot{a}$ = 0. For this to be a must also be equal to 0. However equations (5) and (6) are inconsistent with the conditions on $\dot{a}$ and a: A static universe with ρ > 0 and $\dot{a}$ = 0 can have a=0 if k=+1. However equation (6) for $\rho > 0\ and\ p > 0$ will exclude a from vanishing. Einstein therefore proposed a modification of his equations such that:

$$R_{\mu\nu} - \frac{1}{2}R\ g_{\mu\nu} + \wedge g_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad (7)$$

Where Λ , the cosmological constant, is a new free parameter that will allow a static universe.

Incorporating this modification the Friedmann equations morph into the following:

$$H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2 R_0^2} \qquad (8)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \quad . \qquad (9)$$

For positive spatial curvature and non- negative Λ, $\rho$ and p equations (8) and (9) admit a Static solution which is called 'The Einstein Static Universe'.

When Hubble discovered that the universe is expanding this eliminated the need, (at least[ Empirically), for the Einstein Static Universe [2]. However Λ cannot so easily be eliminated And, in fact, there is a nkon-trivial reason to believe that Λ is non-zero and should be incl-uded in the Einstein Field Equations.

d) The Vacuum Energy

[Λ] = $(length)^{-2}$ where [ ] ≡ the dimensions of. Λ≡ the energy density of the vacuum In particle physics. This identification allows one to consider the scales of the various contr-Ibution to Λ [3],[4].

We write the quantity

$$S = \int d^4x\sqrt{-g}\,[\tfrac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)] \qquad (10)$$

Which is the action of a single scalar field φ, with potential energy V(φ), g is the deter-minant of $g_{\mu\nu}$ , and the energy-momentum tensor is given by:

$$T_{\mu\nu} = \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}(g^{\rho\sigma}\partial_\rho\phi\partial_\sigma\varphi)g_{\mu\nu} - V(\varphi)g_{\mu\nu} \quad . \qquad (11)$$

The configuration with the lowest energy density has $\partial_\mu\varphi = 0$ which leads to:

$T_{\mu\nu}$= -V($\phi_0$) $g_{\mu\nu}$ where $\phi_0$ is the value of φ which minimizes V(φ). Now V($\phi_0$) is not Necessarily equal to zero so that one my write the vacuum energy-momentum tensor as:

$$T_{\mu\nu}^{vac} = -V(\phi_0)g_{\mu\nu} \qquad (12)$$

Or

$$T_{\mu\nu}^{vac} = -\rho_{vac}\, g_{\mu\nu} \qquad (13)$$

where $\rho_{vac}$ ≡ V($\phi_0$), and equation (13) is the only Lorentz-invariant form of $T_{\mu\nu}^{vac}$. Looking at equation (4) we see that if

$$p_{vac} = -\rho_{vac} \quad (14)$$

then the vacuum can be thought of as a perfect fluid.
The Energy-momentum tensor, equation (13), is physically a cosmological constant as can be seen By moving $\Lambda g_{\mu\nu}$ across the equal sign of equation (7) and then setting

$$\rho_{vac} = \rho_\Lambda \equiv \frac{\Lambda}{8\pi G} \quad . \quad (15)$$

Hence we see that the energy-density of the vacuum is the cosmological constant.
It should be noted that we may obtain nonzero vacuum energy without the introduction of Scalar fields. To do so define the action for General Relativity in the presence of a bare cosmo-Logical constant $\Lambda_0$ ,R being the Ricci scalar by:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, (R - 2\Lambda_0) \quad . \quad (16)$$

Extremizing this action ,(with the addition of the appropriatenmatter terms),leads to equations (7).Hence we see that the cosmmmmmmmolomgical constant can be thought of as a constant term in the Lagrangian density of the theory. Equation (16) is therefore a natural starting point for a theory of gravity.

Classically , the effective cosmological constant $\Lambda^{eff}$ is given by :

$$\Lambda^{eff} = \Lambda_0 + V(\varphi) \quad (17)$$

With $\Lambda_0$ defined as in equation (16) and V(φ) being the potential energy (which may change With space and time as the universe passes throoough different phases). Quantum mechanics Adds another contribution to eq uation (17) due to the zero point energies associated with Vacuum fluctuations. As an example we consider an anharmonic oscillator in one dimension With potential energy of form V(x) = $\frac{1}{2}m\omega^2 x^2$. While the zero of the potential energy is at X=0 classically, quantum mechanically the lowest energy is given by $E_0 = \frac{1}{2}\hbar\omega$ (where ħ has Been included here for clarity). In the absence of gravity the vacuum energy of either system (both the classical and the quantum mechanical) is completely arbitrary in that we can add

Any arbitrary constant to the potential energy without changing the physical content of the Theory. However, it should be noted that the zero-point energy must depend on the system Which in this case is the frequency $\omega$. The equivalent situation obtains in field theory [5],[6], [7]. A free quantum field can be thought of as a collection of an infinite number of anharmonic Oscillators in momentum space(the normal modes). Note that the zero point energy of this Infinite collection is infinite. However, the very high normal modes are discarded (we trust our theory only up to a certain ultraviolet cutoff $k_{max}$ making a fundamental blunder in the Calculation for the energy density which is to follow). Proceeding to use this cutoff one then Obtains an expression for the energy density of form:

$$\rho_\Lambda \sim \hbar k_{max}^4 \quad . \qquad (18)$$

Note that in the absence of gravity this term, equation (18), has no effect and is traditionally Discarded by normal ordering. However since gravity does exist in our universe the value of The vacuum energy has very important consequences. Also, it should be noted that the vacuum Fluctuations are quite real as is evidenced by the Casimir effect [8].

2. The conventional wisdom makes a theoretical prediction for $\rho_\Lambda$

The net cosmological constant, from the considerations in section 1, is the sum of a number Of disparate contributions including potential energies from scalar fields and zero-point fluc-tuations of each field theory degree of freedom as well as the bare cosmological constant $\Lambda_0$. Unlike the contribution from $\Lambda_0$ we may make educated guestimates of the magnitudes of the Other contributions: In the Weinberg-Salam model, the phases of broken and unbroken symm-etry have a potential energy difference of roughly $M_{ew} \approx 200$ Gev. The universe is in the broken symmetry phase during our current , low temperature, epoch and is believed to have been in the symmetric phase at significantly elevated temperatures during earlier times. The effective cosmological constant is different in the two phases such that we expect a contribution to the vacuum energy-density currently of order :

$$\rho_{\Lambda}^{ew} \sim (200\,Gev)^4 \approx 3\text{x}10^{47}\,\frac{erg}{cm^3} \quad . \qquad (19)$$

Note that similar contributions can arise without using fundamental scalar fields. For Example Chiral symmetry is believed to be broken in the strong interactions by the non-Zero expectation value of the quark bilinear pair q$\bar{q}$.In this case the energy didfference Between the symmetric and broken phases is of the order of the QCD scale $M_{QCD} \approx$ 0.3 Gev And we would expect a corresponding contribution to the vacuum energy density of order

$$\rho_{\Lambda}^{QCD} \sim (0.3\,Gev)^4 \approx 1.6 \text{ x } 10^{36}\,\frac{erg}{cm^3} \quad . \qquad (20)$$

These contributions are joined by those from any number of,as of yet, unknown phase transitions in the early universe,such as a possible contribution from Grand Unification of order $M_{GUT} \sim 10^{16}$ Gev.

In the case of vacuum fluctuations we choose our cutoff at the energy past which our field Theory can no longer be trusted (the Planck scale) : $M_{PL}$ =$(8\pi G)^{-1/2} \approx 10^{18}$ GeV. Here We expect a contribution of order

$$\rho_{\Lambda}^{PL} \sim (10^{18}\,Gev)^4 \approx 2\text{x } 10^{110}\,\frac{erg}{cm^3} \quad . \qquad (21)$$

Notice that all oataher terms have been discarded in eq (21) which is the conventional Wisdom that is currently in vogue for the theoretical value of the Cosmological constant. Cosmological observations give the result

$$|\rho_{\Lambda}^{obs}| \leq (10^{-12}\;Gev)^4 \approx 2\text{x } 10^{-10}\,\frac{erg}{cm^3} \qquad (22)$$

So that the ratio :

$$\frac{\rho_{\Lambda}^{theo}}{|\;\rho_{\Lambda}^{obs}|} \sim 10^{120} \quad . \qquad (23)$$

For this reason the ratio in equation (23) is called ' the worst catastrophe in theoretical physics'. This remains one of the most significant unsolved problems in fundamental physics.

3. The Conventional Wisdom Versus The New Physics (Two New Approaches)

a) We begin by examining the old approach( ie the conventional wisdom).

The vacuum fluctuations of all naormal modes were thought,(and rightly so), to make a contribution to the cosmological constant. This leads to the following result;

$$\frac{\rho_\Lambda^{theory}}{|\ \rho_\Lambda^{obs}\ |} = \frac{(m_1 + m_2 + m_3 + m_4 + \cdots \ldots\ldots + M_{PL})^4}{(10^{-12} Gev)^4} \approx \frac{M_{PL}^4}{(10^{-12}\ Gev)^4} = \frac{\left(10^{18}\ Gev\right)^4}{(10^{-12} Gev)^4} = 10^{120} \qquad (24)$$

Where, once again, it should be noted that all other modes in equation (24) were discarded Excepting the Planck mass. This is justified, in the conventional approach because the Planck mass Is considered to be practically infinite in the particle physics realm and so perturbation theory Considers it to be infinite in comparison to the other particle masses.

b) The New Physics

i) The first method

The normal modes of the superstring yields the mass eigenstates. Summing over the normal Modes gives a sequence of masses which includes all of the contributions to the Cosmological Constant: $m_1 + m_2 + m_3 + m_4 + \ldots\ldots\ldots M_{PLANCK}$ where $m_1$ is the mass of an,as yet unknown , elementary particle (it, in fact,being the lightest one). Also note that the mass of the primordial magnetic monopole pairs must be,and is, included the mass sum here and in the remainder of this paper [4].

Next we decompose the above sum into integral factors of $m_1$ to obtain the following:

$m_1$ ( 1+2+3+4+5+…+………………..+ ∞ ) where there exists a very large number of $m_i$'s which

In fact is infinite when summed over the normal modes of the superstring.

Let M ≡ the total mass of the string eigenstates such that it can be factorized in the following way:

$$M = m_1 (1 + 2 + 3 + 4 + 5 + \ldots\ldots + \ldots\ldots + \infty) \quad . \tag{25}$$

Originally M was approximated by $M_{PL}$ however,in this new approach, it is clear can't be thrown away. Hence we see that we must reevaluate equation (24) in lite of this new information.

Equation (24) is now replaced by the more accurate result:

$$\frac{\rho_\Lambda^{theory}}{|\ \rho_\Lambda^{obs}\ |} = \frac{m_1^4 (1+2+3+4+\cdots..+\infty)^4}{(10^{-12}\ Gev)^4} \quad . \tag{26}$$

Examining equation (26) we see that the numerator is infinite and in fact the terms in parentheses In the numerator can be expressed as the Riemann Zeta Function:

$$1+2+3+4+5+\ldots\ldots+\infty \equiv \zeta(-1) \quad . \tag{27}$$

Next invoke Zeta function regularization to obtain the result:

$$\frac{\rho_\Lambda^{theory}}{|\ \rho_\Lambda^{obs}|} = \frac{m_1^4}{(10^{-12}\ Gev)^4} (\square(-1))^4 \tag{28}$$

which, after some inspection yields :

$$\frac{\rho_\Lambda^{theory}}{|\rho_\Lambda^{obs}|} = \frac{m_1^4/(-12)^4}{(10^{-12}\ Gev)^4} \quad . \tag{29}$$

To fix $m_1$ we require that

$$\frac{\rho_\Lambda^{theo}}{|\rho_\Lambda^{obs}|} = 1 \quad . \tag{30}$$

Therefore for exact agreement:

$$\frac{m_1^4/(-12)^4}{(10^{-12}\ Gev)^4} = 1 \tag{31}$$

Which yields the result $m_1$ = 0.012 ev.

We identify $m_1$ as the mass of the electron's neutrino $m_{\nu_{e-}}$ since this result is consistent

With current estimates for $m_{\nu_{e-}}$.

Thus the cosmological constants $\rho_\Lambda^{theory}$ and $|\rho_\Lambda^{obs}|$ match exactly if $m_1 = m_{\nu_{e-}}$ =0.012 ev.

It should be noted that this also fixes the mass of the electron's neutrino.

ii) The Second Approach

We consider the RNS Sector where

$$M^4 \equiv \frac{1}{''\alpha'^2}(1+2+3+4+5+\cdots\ldots\ldots.+\infty)^2 \tag{32}$$

Where we have approximated the Planck Mass plus other anomalous terms (ie see [4])

by infinity and by dimensional analysis we identify $\alpha'$ with $\frac{1}{m_1^2}$ so that clearly:

$$M^4 = \frac{1}{\alpha'^2}(\square(-1))^2 = \frac{1}{\alpha'^2}\left(\frac{-1}{12}\right)^2 \quad . \tag{33}$$

Now $\alpha'$ must also be renormalized since it is a mass term to get $\frac{1}{\alpha'^2} = m_1^4\left(\frac{-1}{12}\right)^2$

To get $(m_1)^4 \left(\frac{-1}{12}\right)^2\left(\frac{-1}{12}\right)^2 = \frac{m_1^4}{144^2}$ so that in the RNS Sector:

$$M^4 = \frac{m_1^4}{(144)^2} \quad . \tag{34}$$

This is a fundamental result. It verifies the validity of Superstring theory by making its' long sought after prediction which is valid for currently accessible energies.

In conclusion: We have made the identification of $m_1$ with the mass of the electron's Neutrino and fixed it at $m_1 = m_{\nu_{e-}}$ = 0.012 ev giving exact agreement between the theoretical And observed values of the cosmological constant.

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