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Dirac Magnetic Monopole Pair Production at the Large Hadron

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Abstract : Production rates, as well as the mass spectrum, are given for Dirac Magnetic Monopole Pairs, at the Large Hadron Collider (LHC), having masses $M_{pair} \leq$ 200 GeV for $\sqrt{s}$ =3.5 Tev/beam and $\sqrt{s}$ = 7Tev/beam. These rates are calculated for the initial luminosity L_{in}= 10^{33} cm^{-2} sec^{-1} and the proposed upgraded luminosity L_{high} = 2x10^{34} cm^{-2} sec^{-1}. Dirac Magnetic Monopoles pairs will be produced at the LHC via two possible processes; a) the Drell-Yan Process and b) a new and extremely rare process, which we are proposing here which takes place at these very high energies and predominantly occurs at the higher luminosity L_{high} where energy from P+P→ P+P+ Υ^* is deposited in the interaction region forming sub-nuclear matter, from the destabilized vacuum, which is subsequently trapped in the envelope formed by the combination of the proton beams and the vacuum, hence allowing the process γ*→ M+M to take place. At the high energies at the LHC since we are creating the conditions that were present during the early universe the vacuum reacts to the tremendous amount of energy deposited, in the interaction region, by this process, and tries to envelope it similar to the way that matter fights back when an electromagnetic field propagates through it or an electric circuit fights back by creating resistance when an electric charge flows through it. It is also well known that the bare charge and bare mass of the proton is infinite, necessitating the somewhat artificial procedure of renormalization which may really just be a manifestation of the vacuum having formed an envelope around the bare particle (proton in this case) as a reaction to its infinite charge. A rich harvest of magnetic monopole pairs should

be observed at the above energies and luminosities for $t=6.31x10^7$ sec. specifically we conclude that if 1 in 10^{19} collisions contribute to process b) then for M_{pair}= 200 GeV, $L_{in}=10^{33} cm^{-2} sec^{-1}$ and $t=6.31x10^7$

sec #pairs n_1=1; for L_{in}= (2 to 2.5)$x10^{33}cm^{2}sec^{-1}$. M_{pair}=200GeV and $t=6.31x10^7$sec.

pairs n_1 =3; for M_{pair}= 200 GeV, L_{high} and t (as stated above), # pairs n_1 = 23 with 16 pairs

coming from Drell-Yan and 7 coming from process b); at $\sqrt{s}$= 14 TeV (i.e. 7 TeV/beam), L_{high} and

M_{pair}= 200 GeV, # of pairs n_2 = 8 all coming from process b). Hence we predict that if at $\sqrt{s}$= 7

Tev/beam we observe any magnetic monopoles of mass M_{pair}= 200 GeV, at all, this indicates that the

new process b) has taken place.

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PACS.14.80.HV-Magnetic Monopoles

I. Introduction

When James Clerk Maxwell originally formulated the field equations of electrodynamics he felt that the equations should not only be completely symmetrical in the electric and magnetic fields, but should also be symmetric in the electric and magnetic charge. [1] J.J. Thompson [2] also assumed that magnetic charges exist and he formulated a model of a magnetic charge coupled to an electric charge which has come to be known as "Thompson's Dipole". The total angular momentum per unit volume that Thompson's Dipole has is given by

$$\frac{L}{V}=\frac{3\mu_0}{(4\pi)^2R^3}eg \qquad (1)$$

Where L≡ total angular momentum carried by Thompson Dipole, V≡ volume of sphere centered at field Pt., e≡ electric charge, g≡ magnetic charge.

The total angular momentum L is given by

$$L = \frac{\mu_0}{4\pi} eg \,. \qquad (2)$$

P.A.M. $Dirac^{(3)}$ proposed that the total angular momentum is quantized:

$$\frac{\mu_0}{4\pi} eg = \frac{n\hbar}{2} \qquad (3)$$

Rescaling n

$$eg = n' \frac{\hbar}{2} \qquad (4)$$

And in natural units with $n' = 1$

We arrive at the conventional form of the Dirac quantization condition:

$$eg = \frac{1}{2} \qquad (5)$$

Condition (5) is very useful in the elucidation of the concept of both electric and magnetic charge quantization. Simply speaking if one type of charge is quantized then the other is also.In this paper we will treat magnetic monopole pair production at the LHC for

Proton beams where $\sqrt{s}$= 3.5 Tev/ bean and$\sqrt{s}$= 7 Tev/beams at L_{in} =10^{33}cm^{-2}sec^{-1}and the proposed upgraded luminosity L_{high} = 2x10^{34}cm^{-2}sec^{-1}.

In previous researches we have examined magnetic monopole pair production at the CERN ISR [4] and at BNL's Relativistic Heavy Ion Collider [5] (RHIC) with searches proposed at RHIC to take place in the not too distant future.

We predict that the magnetic monopole pairs produced at the LHC will have masses $M_{pair}\leq$ 200 GeV. Specifically our results for magnetic monopole pair production rates are as follows: a.) for $L_{in}=10^{33}$ cm^{-2} sec^{-1}, M_{pair}= 200 GeV and $\sqrt{s}$=7 TeV and t= 6.31x10^{7} sec, # pairs < 1 and it should be noted that we more than likely will not see a magnetic monopole pair production event (at $pair$ 200 Gev) until L_{in} = (2. To 2.5)x 10^{33} cm^{-2} sec^{-1} where we will then see 2 pairs (# pairs=2); b) at maximal luminosity L_{high}= 2x10^{34} cm^{-2} sec^{-1}, $\sqrt{s}$=7 TeV and t= 6.31x10^{7}sec we should see 16 pairs (#pairs=16) c) at $\sqrt{s}$= 14 TeV (ie.7 TeV/beam) and L_{high}, # pairs= 8 (M_{pair}= 200 GeV) from process b).

The central purpose of this paper is to amplify the information contained in the abstract.

In section 2 we define the distribution functions that we will use for process a) and in section 3 we review and apply the Drell-Yan Mechanism to P+P→ γ*+x where γ*→ M+$\overline{M}$.

In section 4 we introduce a new process P+P→P+P+ γ*, where γ* is anomalous energy deposited in the interaction region ,and calculate the cross section for γ*→ M+$\overline{M}$.

In section 5 the mass spectrum is given for the Drell-Yan and the new process and production rates are tabulated for both processes for the various energies and luminosities. A discussion of the results then concludes this paper.

2. *Distribution functions that we will use.* – We will use the dynamical QCD distributions:[6]

a) Up quark

$$
(2.1)\quad
\begin{cases}
xU_v(x, Q^2) \quad x^\eta \sum_{i=1}^{4} A_i(1-x)^{\eta_i} \\
\eta = 0.5 + 0.0034\,\bar{s} \quad 0.02005\,\bar{s}^2 \\
A_1 = 5.496\,, \qquad A_2 = 3.844\,, \\
A_3 = -0.822\,, \qquad A_4 = 0.450\,, \\
\eta_1 = 3 + 0.798\,\bar{s}\,, \qquad \eta_2 = 5 - 0.384\,\bar{s}\,, \\
\eta_3 = 7 + 34.5\,\bar{s}\,, \qquad \eta_4 = 9 - 3.381\,\bar{s}\,.
\end{cases}
$$

b) Down quark

$$
(2.2)\quad
\begin{cases}
xd_v(x, Q^2) = x^\eta \sum_{i=1}^{4} A_i(1-x)^{\eta_i} \\
\eta = 0.5 - 0.1017\,\bar{s} + 0.0139\,\bar{s}^2\,, \\
A_1 = 2.332\,, \qquad A_2 = 1.907\,, \\
A_3 = -3.657\,, \qquad A_4 = 0.774\,, \\
\eta_1 = 4 + 0.973\,\bar{s}\,, \qquad \eta_2 = 5 \cdot 1.383\,\bar{s}\,, \\
\eta_3 = 7 - 0.389\,\bar{s}\,, \qquad \eta_4 = 9 - 2.042\,\bar{s}\,.
\end{cases}
$$

c) $\bar{u} \simeq \bar{d} \simeq s \simeq \bar{s} = \xi$

$$
(2.3)\quad
\begin{cases}
x\xi(x, Q^2) = A_i(1-x)^{\eta_i} + A_i'(1-x)^{\eta_i'} + B_i \exp[-C_i x]\,, \\
A_i = A_i^{(0)} + A_i^{(1)}\bar{s} + A_i^{(2)}\bar{s}^2\,, \\
\eta_i = \eta_i^{(0)} + \eta_i^{(1)}\bar{s} + \eta_i^{(2)}\bar{s}^2\,, \\
A_i' = A_i'^{(1)}\bar{s} + A_i'^{(2)}\bar{s}^2\,, \\
\eta_i' = \eta_i'^{(0)} + \eta_i'^{(1)}\bar{s} + \eta_i'^{(2)}\bar{s}^2\,, \\
B_i = B_i^{(1)}\bar{s} + B_i^{(2)}\bar{s}^2\,, \\
C_i = C_i^{(0)} + C_i^{(1)}\bar{s} + C_i^{(2)}\bar{s}^2\,.
\end{cases}
$$

d) Gluon distribution

(2.4) $\quad xG(x, Q^2) = A_g(1 \quad x)^{\eta_g} + A_{g'}(1-x)^{\eta_g'} + B_g \exp[-C_g x]$

where for (2.3) and (2.4)

	$x\xi$	xG
$A^{(0)}$	0.0	0.0
$A^{(1)}$	0.0227	0.3125
$A^{(2)}$	−0.0045	−0.0660
$A'^{(1)}$	0.0389	0.9237
$A'^{(2)}$	−0.0011	−0.1021
$\eta^{(0)}$	4.8726	1.4285
$\eta^{(1)}$	0.3428	1.4483
$\eta^{(2)}$	0.1015	−0.0142
$\eta'^{(0)}$	10.9605	1.8769
$\eta'^{(1)}$	1.2669	2.5026
$\eta'^{(2)}$	0.0011	0.4152
$B^{(1)}$	−0.0100	2.7120
$B^{(2)}$	0.0481	−0.0146
$C^{(0)}$	29.9236	29.2319
$C^{(1)}$	3.8858	−3.3769
$C^{(2)}$	1.0451	3.6739

Choosing $\Lambda_{\overline{MS}} = 0.25$, we see that $\bar{s} = 1.11$ The up-quark distributions become

$$xQ_v(x, Q^2) = x^{0.47887}(5.496(1 \quad x)^{3.[illegible]} - 3.844(1-x)^{4.67274} -$$
$$- 0.822(1-x)^{13.[illegible]} + 0.45(1 \quad x)^{3.[illegible]})$$

The down-quark distributions become

$$xd_v(x, Q^2) = x^{0.[illegible]}(2.332(1-x)^{4.[illegible]} + 1.907(1-x)^{6.[illegible]} +$$
$$+ 3.657(1-x)^{6.[illegible]} + 0.774(1-x)^{6.[illegible]})$$

The strange and sea quark distributions become

$$\bar{u} \simeq \bar{d} \simeq s \simeq \bar{s} \equiv \xi$$

$$x\xi(x, Q^2) = 0.01965(1-x)^{5.378} + 0.042(1-x)^{12.372} + 0.0482 \exp[-35.525x].$$

Gluon distribution

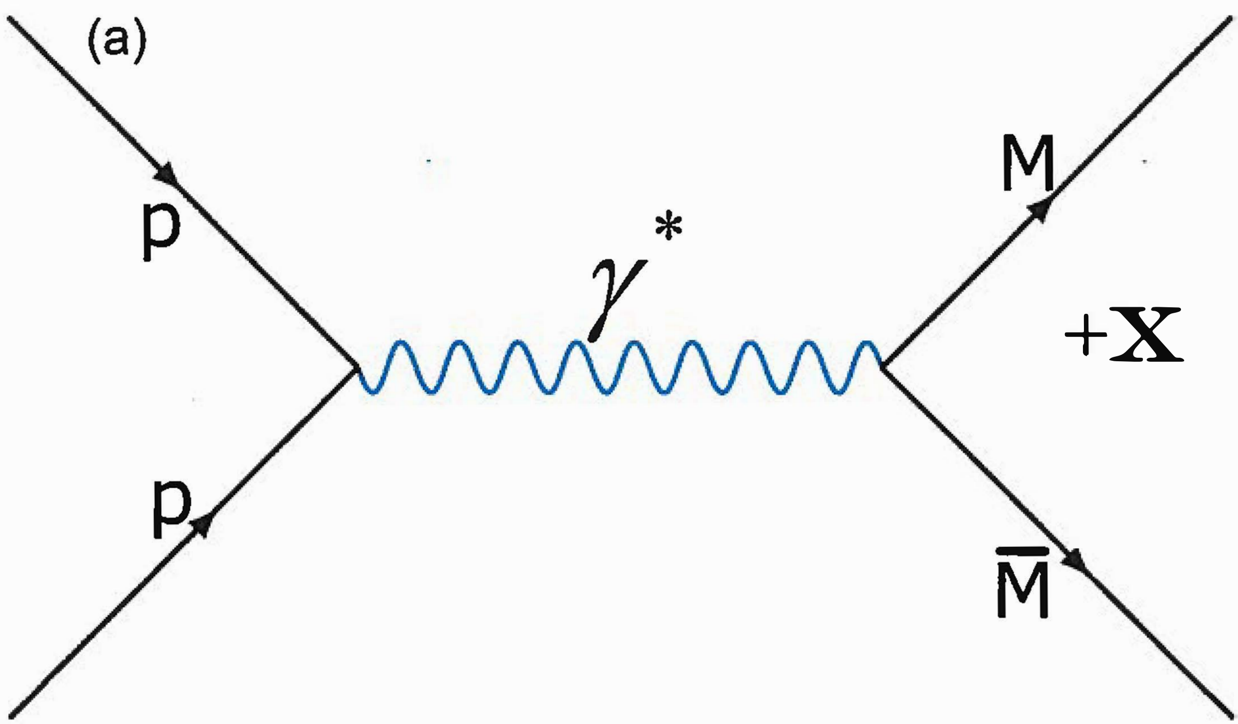
$xG(x, Q^2) = 0.278(1-x)^{3.03} + 0.896(1-x)^{5.2} + 3 \exp[-30x]$

3. The Drell-Yan Process

In past works we have dealt with the Drell-Yan process in P-P collisions and the reader is referred to those *works*[7][8]. Here we will just summarize the results of these works so as not to be redundant. We are considering the process P+P→ γ^*+X where γ^* is a virtual photon which subsequently decays into M+$\overline{M}$ (i.e. magnetic monopole pairs).

The sub-process of relevance to the Drell-Yan mechanism is q+$\overline{q}$→ γ^*+X .

These processes are illustrated in Figure 1.a) and b).



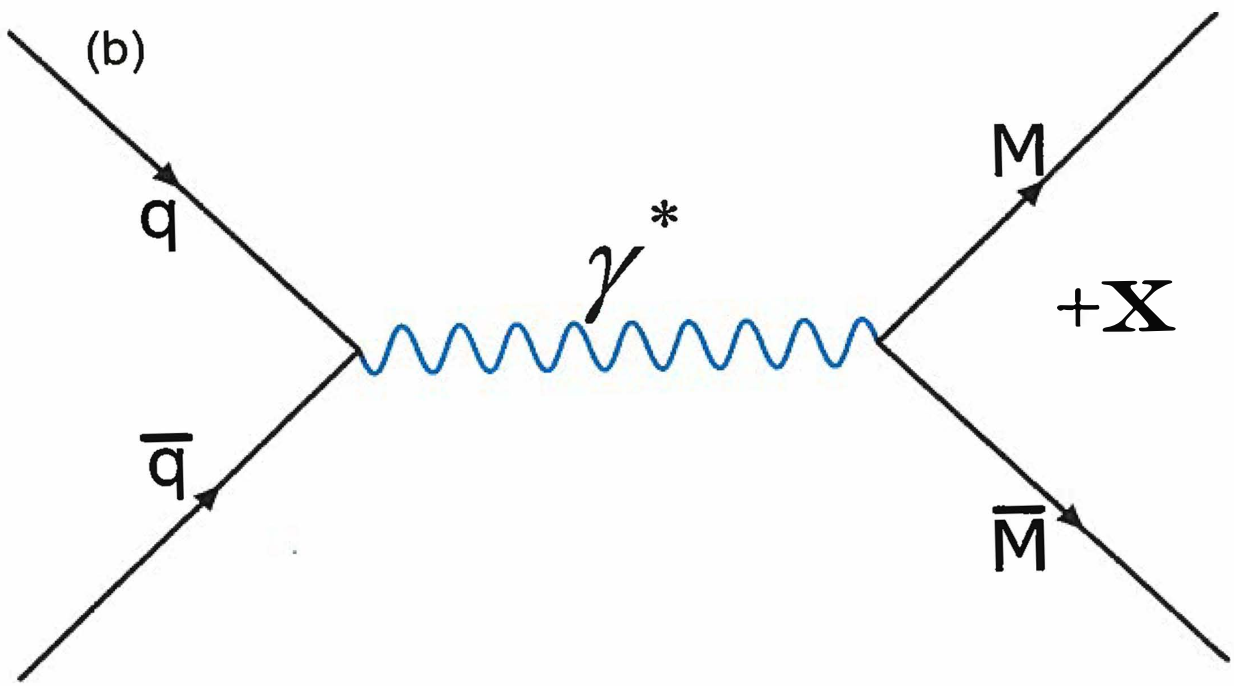


Figure 1. a) The Drell-Yan Process $P + P \to \gamma^* \to M + \bar{M} + X$. Here $P \equiv$ a proton, $\gamma^* \equiv$ a virtual photon, $M(\bar{M}) \equiv$ a Magnetic Monopole(Anti-Monopole) and $X \equiv$ anything ; b) The Drell-Yan sub-process $q + \bar{q} \to \gamma^* \to M + \bar{M} + X$ is very important for Magnetic Monopole pair production. Here $q(\bar{q}) \equiv$ a quark(anti-quark). All other definitions are the same as in figure 1 a)

After some analysis the sub process cross section is given by

$$\frac{d\sigma}{dM^2dy}(q + \overline{q} \rightarrow M + \overline{M}) = \frac{1}{12M^2} e_q^2 \delta(M^2 - S) \qquad (3.1)$$

Which leads to the overall cross-section

$$\frac{d\sigma}{dM^2dy}(p + \overline{p} \rightarrow M + \overline{M} + X) = \frac{1}{12M^4} \iint dx_1 dx_2 \sum_{a=u,d,s} e_a^2 \left[f(x_1)_a^A f(x_2)_{\overline{a}}^B + (x_1 \leftrightarrow x_2)\right]$$

Where $x_1 = \frac{Me^y}{\sqrt{S}}$, $x_2 = \frac{Me^{-y}}{\sqrt{S}}$, y being the rapidity of the pair, and the $f_i^I(x_1 \; or \; x_2)$ are the distribution functions defined in section 2

4. A new, extremely rare, process is hypothesized

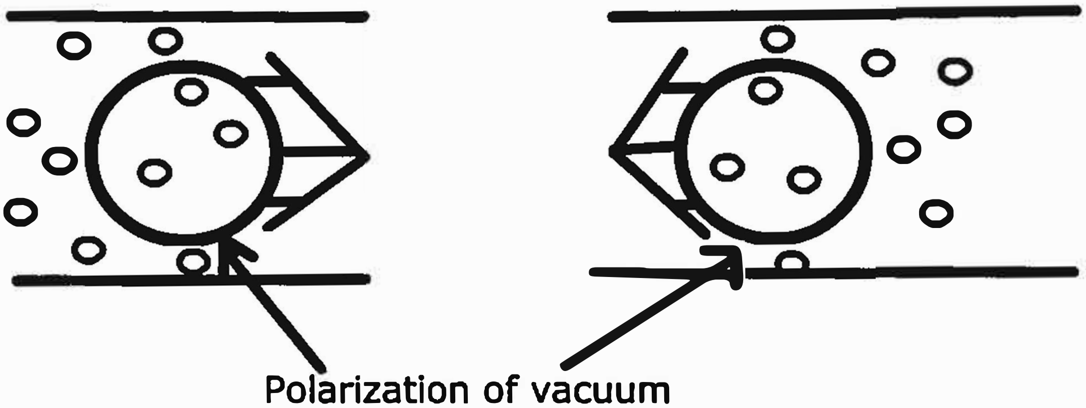
At very high energies we hypothesize that another process becomes important to the overa production rates for magnetic monopole pairs

$$p + p \rightarrow p + p + \gamma * \rightarrow M + \overline{M} \quad \text{where } \gamma^{*} \rightarrow M + \overline{M} \;. \qquad (4.1)$$

Figure 2 gives a schematic of the Roberts model of this very rare process. The basic idea behind this new process is as follows: Consider two very highly energetic proton beams incident upon each other (situation one of figure 2a). Renormalization theory tells us that the bare charge of the proton is infinite and must be shielded by a photon cloud to give us the physical charge which has the value e= 1.6×10^{-19} coul. Looking at situation one, however, notice that the protons are extremely energetic and moving at extremely high speed through the physical vacuum which, we hypothesize, tends to form tubes around the protons to impede the flow of charge through it. The vacuum polarizes due to the tremendous rate of charge flowing thru it. Also note that in figure 2a) the region in front of the two protons is not yet connected by the flux tubes since charge has not entered that region. In figure 2b) which is situation two, note that the flux tubes formed by the flow of charge, form the proton beams, and the breakdown of the vacuum have connected. The sub-nuclear matter due to the vacuum polarization is also clearly within the flux tube.

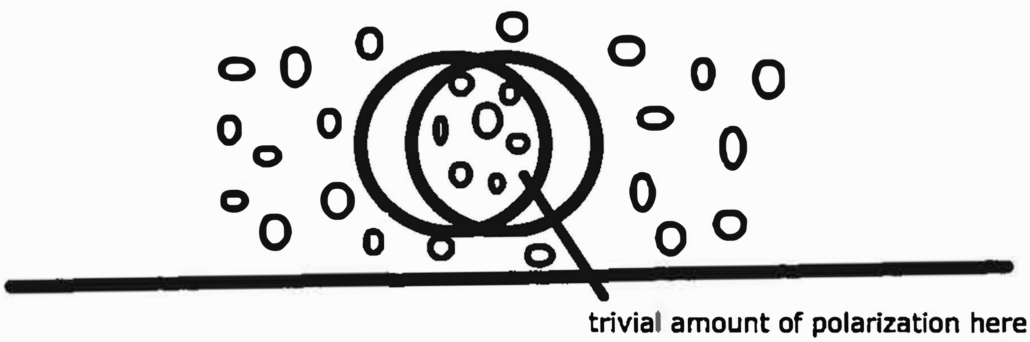
Situation one

a)



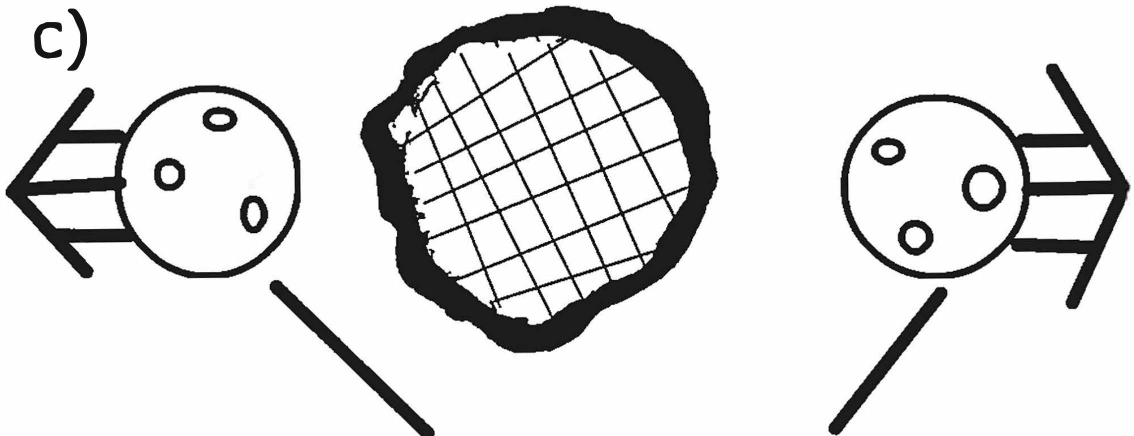
b)

Situation two



Situation three

c)



Note:
Exiting protons are much less energetic
And the flux tubes around them aren't
necessary, their function having been
taken over by the photon cloud

D) Situation four

$$\gamma^* \rightarrow M + \bar{M} + X$$

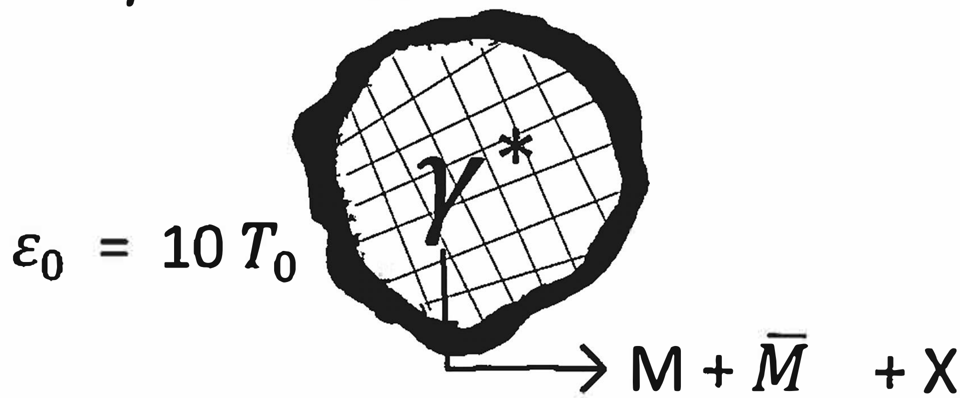


Figure 2 Schematic of the newly theorized process. a) Situation One. Ultra high energy protons are incident upon each other. The dynamical vacuum polarizes to form tubes due to the ultra high rate of flow of bare charge; b) Situation Two. The tubes have become one tunnel and the penetration of the quarks of one proton into the other gives a negligible contribution to the polarization; c) Situation Three. As the protons leave the interaction region the flux tube closes around the largely contained and moderately energetic quark-gluon plasma forming an envelope; d) Situation Four. The process $\gamma^* \rightarrow M + \bar{M} + X$ takes place in the new interaction region. If this newly theorized process occurs in 1 in 10^{14} collisions an observable signal will occur at $\sqrt{S} = (7 + 7 = 14)$Tev

In this model (The Roberts Model) the flux tube serves the purpose of shielding the rest of the universe from the proton's bare charge. This is necessary only when other production mechanisms (such as Drell-Yan) are not present. When other mechanisms are present the tubes dissolve, no longer being necessary.

Next consider figure 2c) (situation three):

Here the protons are exiting the interaction region and are much less energetic so that normal vacuum processes are taking place via the photon clouds familiar from Renormalization Theory. The flux tubes close around the subnuclear matter left from the vacuum polarization forming an envelope around the new interaction region, ever tightening,

allowing the process $\gamma * \rightarrow M + \overline{M} + X$ to take place.

If the process outlined in this section takes place in 1 out of 10^{19} collisions, (it is very rare indeed), then magnetic monopoles should be observed at $\sqrt{S}^{total} \approx (7+7)$ Tev or 14 Tev.

Calculation of cross-section for $\gamma* \rightarrow M + \overline{M} + X$

In this process $\gamma*$ is the anomalous energy deposited in the interaction region largely from the vacuum polarization process and marginally from the proton beams (since the protons are moving too quickly to deposit much energy). From previous *researches*(4,5,7,8) in order to produce magnetic monopoles having $M_{pair} \leq 200$ GeV then $T_0 \approx 4$ GeV and $\epsilon_0 \leq 40\ ^{Gev}/_{fm^3}$ (see chart #1) where I am modeling this production process by thermal production. The Thermal Production cross-section is given by:

$$\sigma_{thermal}(\gamma^* \rightarrow M + \overline{M} + X) \approx 15.2 \frac{M^2}{X_F} \exp[\frac{-X_F}{2T_0} \frac{E_{max}}{\gamma_B}]\ (Gev^{-2}) \quad (4.2)$$

Note : Here $\epsilon_0 = 10\, T_0$, $X_F = 1.6$, $E_{max} = 30$, $\gamma_B = \frac{\gamma_0}{2\gamma_F}$, $\gamma_0 = 30$ and $\gamma_F = 2$.

When these values are used this yields the following results:

$$\sigma_{thermal}(\gamma^* \rightarrow M + \overline{M} + X) = \Lambda^{tube}\, 6.63 \times 10^{-23}\, cm^2 \quad (4.3)$$

Where $\Lambda^{tube} \equiv$ probability for flux tube to form.

However it should be noted this is an extremely rare process which we hypothesize to take place only in 1 out of 10^{19} collisions. Hence we see that $\Lambda^{tube} \approx 10^{-19}$.

Therefore we see that at LHC energies our guestimate gives :

$$\sigma_{thermal}(\gamma^{*} \rightarrow M + \overline{M} + X) = 6.63 \times 10^{-42}\ cm^2 \qquad (4.4)$$

and should solve the dessert problem for magnetic monopole production at higher energies. It should also be noted that Λ^{tube} should increase with $\sqrt{S}^{total}$. Large mass muon pairs should also obey this effect. At lower energies conventional renormalization effects are able to keep pace with the protons in the proton beams, but aren't able to do so at the extreme energies of the LHC, thus necessitating the new process.

The number 10^{-19} is of fundamental significance to the new process because it has another physical interpretation: The Electromagnetic Force Interaction Time, which we interpret to be the formation time of the photon cloud, is t_{em}=10^{-19} sec. Hence we see that according to our flux tube formalism a flux tube must form in under 10^{-19} sec or the photon cloud will form.

For the electromagnetic force, in our universe, to turn on it must be renormalized by the photon cloud. So at energies present at the LHC 10^{-19} sec is the time for the photon cloud to form and the electromagnetic force therefore to be switched on. The question is, then, is it possible for the flux tubes, necessary for process b), to form? To examine this question the cross-section of a flux tube is $\sigma_{flux\ tube}$= $1.25 \times 10^{-15}\ cm^2$ and L_{in} = $10^{33} cm^{-2}$ sec , and we will use the following values: L_{High}≈ $2 \times 10^{34} cm^{-2} sec^{-1}$ and $t_{em} \leq 10^{-19}$ sec. Now $n_{flux\ tube}$≡ the number of flux tubes created And is given by:

$$n_{flux\ tube} = \sigma_{flux\ tube} \int L\, dt_{em} \qquad (4.5)$$

Now $n_{flux\ tube} \approx 0$ for L_{in} even when $t_{em} = 10^{-19}$ sec. At L_{High} and for $t_{em} = 4.0 \times 10^{-20}$ sec

$n_{flux\ tube} = 1$.

In the first case the photon cloud will be produced pre-empting the flux tube, while in the 2nd case (at L_{High}) the flux tube will be created with 6.0 x $10^{-20} sec$ to spare. It should be noted that in this model the existence of the flux tube and photon cloud are mutually exclusive. While one exists the other cannot. We conclude therefore that at LHC energies and L_{High} , flux tubes will be created making process b.) possible.

5. Results , Conclusion and Discussion

The results for the mass spectrum for Drell-Yan and production rates for both Drell-Yan and the new process b) are given in the table below.

In this table:

$n_1 \equiv$ # magnetic monopole pairs produced assuming L=L_{in}= 2.5x10^{33} cm^{-2} sec

n_2= magnetic monopole pairs produced assuming L=L$_{high}$ 2.0x10^{34} cm^{-2} sec

where t=6.3x10^{7} sec is running time for beams.

*in the n_1 and/or n_2 column indicates particle production from process b.)

From the table below we see that at high energies and large masses production rates and the mass spectrum fall off dramatically. The new process P+P→P+P+ γ^* → M + $\overline{M}$ + X

Gives results at these high energies that Drell-Yan does not, leading us to the conclusion that if we see any magnetic monopole pairs of $M_{pair} \approx 200\ Gev$ then they must have come from the anomalous process b).

Mass Spectrum (Drell-Yan and Anomalous Process)

$\sqrt{S}$ = (3.5 + 3.5) Tev=7 Tev

M_{pair}(Gev)	C.S. (cm^2)	n_1	n_2
10	5.13×10^{-39}	**	**
20	8.97×10^{-40}	142	1130
30	5.13×10^{-40}	81	646
40	3.21×10^{-40}	50	404
50	2.05×10^{-40}	32	258
60	1.41×10^{-40}	22	177
70	1.05×10^{-40}	16	132
80	8.08×10^{-41}	12	101
90	6.28×10^{-41}	10	79
100	5.13×10^{-41}	8	65
110	4.24×10^{-41}	7	53
120	3.56×10^{-41}	6	45
130	2.95×10^{-41}	4	37
140	2.62×10^{-41}	4	33
150	2.28×10^{-41}	4	29
160	2.0×10^{-41}	3	25
170	1.77×10^{-41}	3	22
180	1.60×10^{-41}	3	20
190	1.42×10^{-41}	2	18
200	1.30×10^{-41}	2 (1*)	16 (7*)

$\sqrt{S}$ = (7 + 7) Tev= 14 Tev

M_{pair}(Gev)	C.S. (cm^2)	n_1	n_2
10	1.29×10^{-40}	20	162
20	2.25×10^{-41}	4	28
30	1.29×10^{-41}	2	16
40	8.04×10^{-42}	1	16
50	5.14×10^{-42}	1	6
60	3.54×10^{-42}	0	4
70	2.64×10^{-42}	0	3
80	2.03×10^{-42}	0	2
90	1.58×10^{-42}	0	2
100	1.29×10^{-42}	0	1
110	1.06×10^{-42}	0	1
120	8.94×10^{-43}	0	1
130	7.4×10^{-43}	0	1
140	6.60×10^{-43}	0	1
150	5.72×10^{-43}	0	1
160	5.02×10^{-43}	0	1
170	4.44×10^{-43}	0	1
180	4.0×10^{-43}	0	1
190	3.57×10^{-43}	0	0
200	3.33×10^{-43}	0 (1*)	0 (8*)

(**) Ruled out by previous searches so the results here are spurious.

Table of Results. This table contains the mass spectrum for M_{pair} for the Drell-Yan process and the Production rates for the newly theorized process. Here $n_1 \equiv$ Magnetic Monopole pairs produced Assuming L= L_{in} = $2.5 \times 10^{33} cm^{-2} sec^{-1}$, $n_2 \equiv$ # Magnetic Monopole pairs produced assuming that L= L_{High} =$2.0 \times 10^{34} cm^{-2} sec^{-1}$ with t $\equiv$ running time of beams= 6.3×10^{7} sec being in common to n_1 and n_2. Asterisked (*) quanties denote the presence of process b).

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