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Microwaves from Extra Galactic Radio Sources Found to Deflect at Minimum Impact Parameter ξ Corresponding to the Solar Radius R

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ABSTRACT

Findings show that the gravitational deflection of electromagnetic waves in the microwave frequency spectrum are severely impact parameter dependent at the plasma limb of the sun. By definition the **impact parameter** ξ is the nearest point of approach of a given ray of light or a ray of microwaves to the center of the gravitating mass M that is inclosed in an analytical Gaussian sphere of radius R. The light bending rule of General Relativity predicts that **impact parameters** of $\xi \geqq R$ for gravitationally bent rays of light and microwaves should occur in empty vacuum space as well as in the plasma limb of the sun, where R is the radius of the analytical Gaussian sphere that encloses the gravitating mass M of the sun. The past century of astrophysical observations show that the bulk of gravitational light bending effects has been observed primarily at the plasma limb of the sun, namely, at impact parameters of $\xi \approx R$. With current technical means in Astrophysics, the gravitational light bending effect should be an easily observable effect for impact parameters corresponding to several solar radii above the plasma limb of the sun, namely, at $\xi = 2R$, $\xi = 3R$, $\xi = 4R$, etc., etc., at $\xi = nR$, for analytical Gaussian spheres of several solar radii R. The corresponding effects of gravitational deflection should be 1/2, 1/3, 1/4, ..., 1/n times 1.752 arcsec observed at the solar plasma limb. Of course, this assumes the light bending rule of General Relativity applies to all empty vacuum space above the surface of the sun as well as in the plasma limb. Findings show that the plasma atmosphere of the sun represents an *indirect interaction* between the gravitational gradient field of the sun and the microwaves from the extra galactic radio pulsar sources. A minimum energy path calculation, supporting this argument, leads to a derivation of the very same light bending equation that was obtained using the assumptions of General Relativity. The measurement on the gravitational deflection of microwaves at the Solar plasma limb by the researchers *Lebach et al.*[10] and a number of other researchers used very-long-baseline-interferometer (VLBI) techniques and various types of synthesis telescopes. The measurements consistently observed to be at minimum impact parameters corresponding to the solar plasma limb of the sun appears to confirm the minimum energy path calculation for the deflection of microwaves propagating in the solar plasma limb. PACS: 95.30Sf, 04.25.dg, 52/25/Qt. 52.40.Db

1 Introduction

We shall examine the evidence for gravitational lensing in our region of space near to us, starting with the nearest star to us, our sun. The light bending rule of General Relativity predicts that a *direct interaction* takes place between the gravitational field of the lensing mass and the rays of light from the stars. The light bending rule of General Reltivity predicts that a *indirect interaction*takes place between the gravitational field of the lensing mass and the rays of light from the stars. *Einstein A.*[7] The past century of solar light bending effects were observed primarily at the thin plasma limb of the sun, namely, at impact parameters of $\xi \approx R$. This argument is strongly supported by a calculation which led to a derivation of the very same light bending equation obtained by General Relativity. *Dowdye*[6] The derivation assumes a minimum energy path of waves propagating in the solar plasma atmosphere exposed to the gravitational gradient field of the sun. The researchers *Lebach et al.*[10] made VLBI observations on extra galactic radio sources to determine the gravitational deflection of the microwaves from these sources. Microwaves were observed to deflect only at the plasma limb of the sun. The researchers reported a gravitational deflection of 0.9998 +/- 0.0008 times 1.752 arcsec which were the most accurate of the measurements. A number of other researchers, *Weiler*[18], *Weiler*[19] and *Riley*[15] used various types of synthesis telescopes to measure the microwave deflections. Appendix A gives a detail calculation for the gravitational deflection of electromagnetic waves in the solar plasma limb based on a minimum energy path, also from the Reference *Dowdye*[6]. Examining the lower boundary of the solar atmosphere and the plasma-free vacuum space several solar radii above the limb of the sun, the solar light bending effect acting on the rays of starlight appear to deviate from the predicted $1/R$ effect of the light bending rule of General Relativity. A close study of the stars in our own region of space, less than hundreds of light-years away, appear to exhibit the very same gravitational light bending effects as that of our nearest star, the sun. There are many cases whereby likely gravitational lenses and light sources are just by chance co-linearly aligned with earth based observers, presenting vast opportunities for the observation of Einstein rings as is predicted by the light bending rule of General Relativity. Thus, the images of the Einstein rings should be ever present in the star-filled skies. It should be no surprise, however, that the Einstein rings are **not** observed in the star-filled night skies as the required impact parameters would have to be such that the rays of starlight propagate well above the plasma limb of the potential stars. Any rays of starlight responsible for conveying the images of the Einstein rings to our telescope must propagate at astronomical distances in plasma-free space well above the plasma limb of potential lensing stars. Any potential for a gravitational deflection must occur in the plasma-free space significantly above the plasma limb of a potential lensing star. A gravitational deflection in that case would require a *direct interaction* between gravity and electromagnetism as is predicted by the light bending rule of General Relativity. Given the mean astronomical distances between the stars in our space, in the order of **light-years**, the failed observation of Einstein rings in our star-filled skies are primarily due to the **very short plasma limb focal length** of potential lensing stars, in the order of **astronomical units (AU,s)**. Note: Our sun is but a typical lensing star and has a **plasma limb focal length** of just **565 AU's**.

2 The Important Fundamentals

An application of Gauss's law, applied to gravitation as well as to electromagnetism along with the principle of optical reciprocity clearly show that a co-linear alignment of the observer, the lens and the source is unnecessary for an observation of a gravitational light bending effect, as predicted by the light bending rule of General Relativity. The gravitational effect at the surface of an analytical Gaussian sphere due to the presence of a point-like gravitating mass that is enclosed inside of the sphere depends only on the quantity of mass enclosed. The size or density of the enclosed mass particle is not important. (Dowdye[5]) (Dowdye[6]) The Gauss' law of gravity (see, e.g., Arfken[3], Jackson[9]) is a Mathematical Physics tool that encloses a gravitating mass particle inside of an analytical Gaussian sphere of radius R. The gravitational field at the surface of this sphere depends solely on the mass M that is enclosed. An analogy to this principle encloses an electrically charged particle inside of a Gaussian surface in application to the electric field of the charged particle in the discipline of Electromagnetism (Jackson[9]). The principle of optical reciprocity (Born[2], Potton[14]) simply states that the light must take the very same minimum energy path or least time path, in either direction between the source and the observer. This fundamental principle is an essential tool for the understanding of complex lensing systems in Astronomy and Astrophysics (Carroll et al.[3]).

2.1 Gauss Law applied to a Point-Like Gravitating Mass

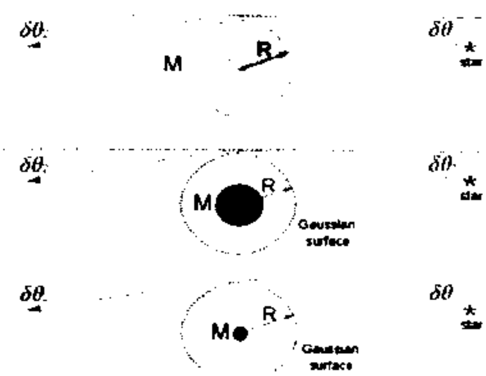

Fig. 1 Gauss' Law is applied to Equal Gravitating Masses of Different Radii of Mass-Spheres enclosed within Spherical Gaussian Surfaces of the same radius R. This important principle graphically demonstrates that the gravitational deflection is dependent on the mass M but is independent of the density of the enclosed mass. In each of the 3 cases illustrated above the impact parameter ξ=R is the same.

Any gravitational effect acting on a light ray due to the presence of a gravitating point like mass displaced by the distance of the impact parameter of $\xi = R$ theoretically depends on the amount of Mass M that is enclosed within the analytical Gaussian sphere of radius R as illustrated in Figure 1. Any gravitational effect that would be noted at the surface of the analytical Gaussian sphere should in principle be totally independent of the radius of the mass particle or the density of the mass that is enclosed within the Gaussian sphere of radius R. From Gauss's Law (Equation 2) equal masses of different radii will theoretically have equal gravitational effects at the surface of the Gaussian sphere. The light bending rule

$$\delta\theta = \frac{4GM}{Rc^2} \tag{1}$$

of General Relativity is essentially a localized $\frac{1}{R}$ effect. We are dealing with astronomical distances. Thus, the bulk of the gravitational effect on the path of particles of light would takes place along a segment of the light ray that encloses the impact parameter ξ. This segment may be only several orders of magnitude greater than the radius R of the Gaussian sphere that encloses the gravitating mass M. The predominant effect of the gravitational field on the bending of the light ray would occur along this short segment of the light path, maximizing at the point where the light ray is tangent to our analytical Gaussian sphere, namely, at the impact parameter $\xi = R$. A Mathematical Physics tool known as Gauss' law, Arfken[2], Jackson[9], is examined. Gauss's law (Equation 2)

$$\int_S \boldsymbol{g} \cdot d\boldsymbol{A} = -4\pi GM \qquad (2)$$

is applied directly to the gravitating masses where the gravitational field $\boldsymbol{g}$ is a function only of the mass M enclosed by the spherical Gaussian surface S. The gravitational flux at the surface of the analytical Gaussian sphere is totally independent of the radius R of the sphere. (Born[2]) The idea here is that the gravitational field at this analytical Gaussian surface is only a function of the mass that it encloses. (Arfken[2], Jackson[9]) Any mass M, regardless of the radius of the mass particle that is enclosed inside of the Gaussian spherical surface of radius R will contribute exactly the same gravitational potential at the Gaussian surface. In Figure 1, the gravitational field points inward towards the center of the mass. Its magnitude is $g = \frac{GM}{R^2}$. In order to calculate the flux of the gravitational field out of the sphere of area $A = 4\pi R^2$, a minus sign is introduced. We then have the flux $\Phi_g = -gA = -(\frac{GM}{R^2})(4\pi R^2) = -4\pi GM$. Again, we note that the flux does not depend on the size of the sphere. It is straightforwardly seen that a direct application of Gauss's law to the light bending rule, Equation 1, coupled with the essential principle of *optical reciprocity* (Potton[14]), removes any requirement for a co-linear alignment of the light source, the point-like gravitating mass particle (the lens) and the observer for observation of a gravitational lensing effect as suggested by General Relativity. (Dowdye[5]) From Equation 2, the flux of the gravitational potential at the surface of the Gaussian spheres, as illustrated in Figure 1, is the same for all enclosed mass particles of the same mass M, regardless of the size of the mass particle. As a result, each mass particle will produce the very same gravitational light bending effect $\delta\theta = \delta\theta_1 + \delta\theta_2$, where $\delta\theta_1$ and $\delta\theta_2$ are the bending effects on the ray of light on approach and on receding the lens, respectively. This of course assumes the validity of Equation 1. This symmetry requirement suggests that $\delta\theta_1 = \delta\theta_2$. From Equations 1 and 3 it follows that $\delta\theta = 2\delta\theta_1 = \frac{4GM}{Rc^2}$ and $\delta\theta_1 = \delta\theta_2 = \frac{2GM}{Rc^2}$. This says that the total contribution of the light bending effect due to the gravitating point-like mass particle on any given infinitely long light ray is theoretically divided equally at the impact parameter $\xi \approx R$, separating the approaching segment and the receding segment of the optical path. A confirmation of this will be clearly seen later with application of the *principle of reciprocity* and a demonstration of a simple derivation of the equation of the Einstein ring, illustrating the *symmetry requirement* of General Relativity.

2.2 Optical Reciprocity applied to the Lensed Light Ray

In any space, the *principle of reciprocity* (Born[2]),(Potton[14]), a very fundamental principle of optics, must hold as illustrated in Figure 2. The principle simply states that any photon or wave of light moving on a **preferred optical path**,

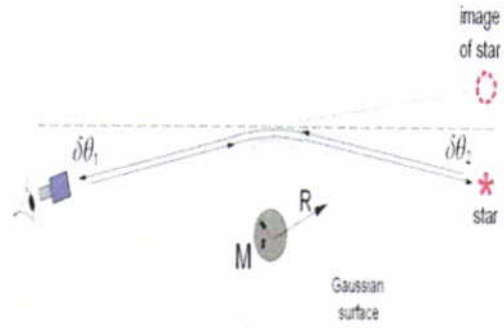

Fig. 2 The Fundamental Principle of Optical Reciprocity Illustrated on a hypothetically Gravitationally Lensed Light Ray shows that the path of a Gravitationally Lensed Light Ray will be the same in both directions, from the source to the observer and from the observer back to the source.

from the source to the observer, must take the very same optical path from a hypothetical laser gun of the observer back to the source. As a consequence of this fundamental principle, any additional sources placed along the same preferred optical path will all appear to the observer to be located at the very same image position of the most distant source. As a consequence of this principle, all light emitting sources on a single preferred optical path will appear to the observer to be co-located at the very same point, appearing as a single light emitting source. This scarcely mentioned fundamental *principle of optics* is directly applicable to the Astrophysics at the galactic center. The total gravitational light bending effect acting on the light ray upon approach and upon receding a point-like gravitating mass is give by

$$\delta\theta = \delta\theta_1\Big|_{\substack{approaching\\ the\ lens}} + \delta\theta_2\Big|_{\substack{receding\\ the\ lens}} = \frac{4GM}{Rc^2} \qquad (3)$$

In this example the gravitating mass M is chosen to be positioned at the midpoint on the line joining the observer and the light source for the simplified special case $D_L : D_{SL} : D_S = 1 : 1 : 2$. (Narayan[13]) This simplified special case is illustrated in Figure 3.

The astronomical distance D_L is the distance from the observer to the lens, D_{SL} is the distance from the lens to the source and D_S is the distance from the observer to the source. Also again, we note that this case is a simplified special case, where $D_L = D_{SL}$, presented in most academic textbooks. There is no requirement at all that the lens be positioned exactly at the midpoint for an observation of a theoretical Einstein ring. See Appendix B for the general case ($D_L \neq D_{SL}$). The vast astronomical distances between the stars most assuredly would present much larger impact parameters of ξ much greater than the radius of the lensing stars under observation, on a much grander scale to any bent light ray passing by the lensing stars. An *indirect interaction* between light rays and the gravitational field of the more distant stars could not occur in the plasma-free vacuum space. The mean astronomical distances in the

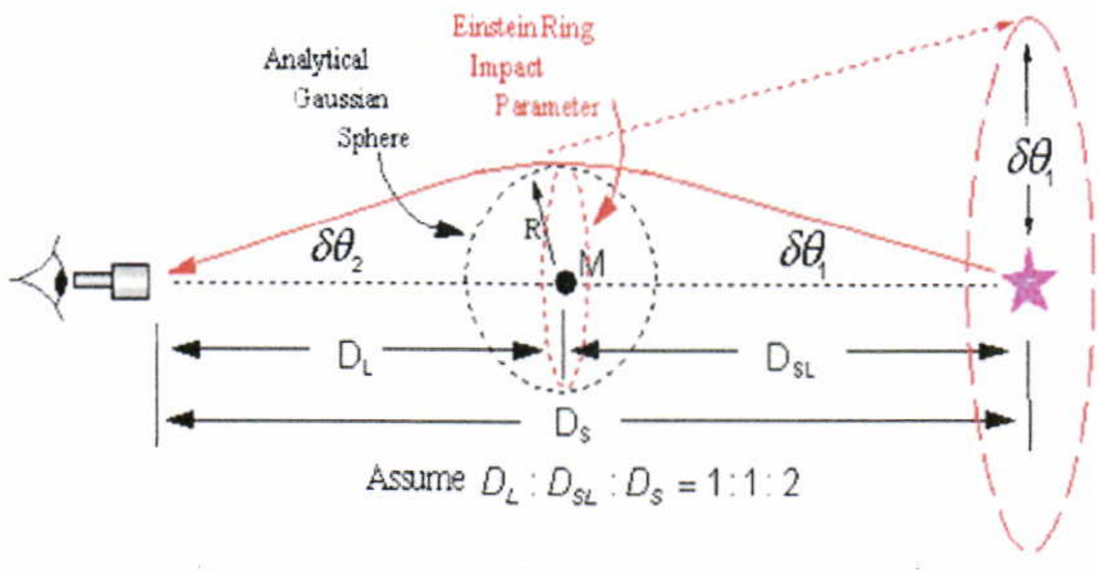

Fig. 3 A Symmetry Requirement for the assumption of the accumulative lensing effect using predictions according to the Light Bending Rule of General Relativity. A gravitation mass M is chosen to be located at the midpoint on a line between the light source and the observer. D_L is distance between the observer and the lens. D_{SL} is distance between the lens and the source. D_S is distance between the observer and the source. This is the simplified special case $D_L = D_{SL}$ presented in most academic textbooks.

celestial skies and consequently the much larger impact parameters ξ, much greater than the radius of a lensing star where the gravitational bending of electromagnetic waves may occur by means of an *indirect interaction* at the stellar plasma limb of a lensing star, may be the most fundamental reason for the failed observation of Einstein rings in the star-filled skies.

2.3 Einstein Ring Equation derived from Symmetry Requirements Assumption

From symmetry we have

$$\delta\theta_1 = \delta\theta_2 = \frac{2GM}{Rc^2} \tag{4}$$

Again, the astronomical distance D_L is the distance from the observer to the lens. Since we are dealing with very small angles, from Figure 3, the deflection of the light ray due to the gravitational effect on approach to the gravitating mass is simply $\delta\theta_1 = \frac{R}{D_L} = \frac{2GM}{Rc^2}$ wherefrom $\frac{R^2}{D_L} = \frac{2GM}{c^2}$ and $\frac{R^2}{{D_L}^2} = \frac{2GM}{D_L c^2} = {\delta\theta_1}^2$. Solving this for the radius of the impact parameter of the light ray and thus the radius of the Einstein Ring expressed in units of radians we have

$$\delta\theta_1 = \sqrt{\frac{2GM}{D_L c^2}} \tag{5}$$

which is the radius of the Einstein ring in units of radians for a lens place exactly midway between the source and the observer. This is a special case, where $D_L = D_{SL}$. (See Appendix B for the general case ($D_L \neq D_{SL}$)) Note that the gravitational bending effect on the light ray for the approach segment alone is exactly equal to the radius of the solved Einstein ring expressed in radians and is given as

$$\delta\theta_1 = \frac{2GM}{Rc^2} \tag{6}$$

This effect is exactly one half of the total accumulative gravitational effect acting on the light ray for the approach and receding segments. (Dowdye[5]) This principle, an essential Mathematical Physics principle on lensing, is often totally missed by researchers attempting to deal with this topic. From symmetry requirement, the integral gravitational effect on a light ray upon approach to a gravitating mass positioned exactly at the midpoint of a line joining the source and the observer, must equal that of the integral gravitational effect on the light ray upon receding the gravitating mass

$$\delta\theta_1\big|_{\substack{approaching \\ the\ lens}} = \delta\theta_2\big|_{\substack{receding \\ the\ lens}} \tag{7}$$

as suggested by Equation 4 and the laws of conservation of energy and of momentum. (Dowdye[5]) This is a rarely covered fundamental on gravitational lensing in the textbooks. The accumulative gravitational effect along the light ray must sum the total effects of gravity acting on the light ray for both the approach and receding segments of any ray of light passing by a point-like gravitating mass. (Dowdye[4]) The total light bending effect is therefore

$$\delta\theta = \frac{4GM}{Rc^2} \tag{8}$$

In all cases, the fundamental principle of optical reciprocity must hold. This is a given. The principle of optical

Table 1 Astrophysical Data of the Sun

Solar Mass	M	$1.99 \cdot 10^{30} Kg$
Solar Radius	R	$6.96 \cdot 10^{8} m$
G Constant	G	$6.67 \cdot 10^{-11} m^3 s^2/Kg$
Velocity of Light	c	$2.99792458 \cdot 10^{8} ms^-1$
$\delta\theta$ (rad)	$\frac{4GM}{Rc^2}$	$8.4952 \cdot 10^{-6}$ rad
$\delta\theta$ (deg)	$\frac{4GM}{Rc^2}$	0.0004867 deg
$\delta\theta$ (arcsec)	$\frac{4GM}{Rc^2}$	1.752 arcsec
Radius of Sun	$R(deg)$	0.275 deg
Focal Length	$\frac{R(deg)}{\delta\theta(deg)}$	565.0 AU

reciprocity simply states that any light ray or a photon of light must take the very same path, along the same minimum energy path, in either direction between the source and the observer as depicted in Figure 2. Using the light bending rule of General Relativity, it is straightforwardly and theoretically demonstrated that all observers of varying distances from a gravitating mass or lens should see an Einstein ring. Only a mid-field observer, one who is placed such that the lens is exactly mid way between the source and the observer, will derive Equation 6. This equation gives exactly the same numerical value as that given by Equation 5 for a simplified special case, where $D_L = D_{SL}$. This simple case is presented in most academic textbooks. Appendix B gives the general case where D_L is **not** necessarily equal to D_{SL}. In the general case, the lens **may not** be placed exactly midway between the light source and the observer. The near-field observer, one who is near to the lens, and the far-field observer, one who is far from the lens, both will derive Einstein ring equations with coefficients corresponding to their unique geometries. Each observer has distinct sets of lensed light rays, each lensed light rays with their corresponding axis of symmetry. A light ray that is gravitationally bent by a point-like gravitating mass, as predicted by General Relativity, will always have an axis of symmetry associated with it. The axis of symmetry will be perpendicular to the line joining the source and the observer only when the lens is positioned exactly at the midpoint on the line joining the observer and the source. Theoretically, all observers should see, according to the light bending rule of General Relativity, an Einstein ring. (Dowdye[5]) This essential key point is missed in all too many lectures on this subject matter.

2.4 Condition for Observing of an Einstein Ring using a Lens of 1 Solar Mass and 1 Solar Radius

Using the collected astrophysical data and the astrophysical constants from Table 1, we find that a stellar system that has 1 solar mass and 1 solar radius, Equation (8) yields a light bending angles of $8.4952 \cdot 10^{-6}$ radians. This angle is 0.0004867 degrees or 1.752 arcsec. The diameter of the solar disk is observed to be 0.55 degrees, a radius of 0.275 degrees. If the radius of the solar disk were compared with the angle of solar light bending of the plasma limb (in degrees), we would have a factor of $\frac{R(deg)}{\delta\theta(deg)} = 565.0$. This means that in order to observe an Einstein ring of a distant stellar light source due solely to the plasma limb of the sun, the observer would have to back away from the sun for at least 565 mean Earth orbital radii or astronomical units(AU's). This is the **focal length of the plasma limb lensing system** of the sun. It is that distance required for the parallel rays of starlight to converge to a point after being deflected by the solar plasma limb. If the observer were to back off to a distance greater than 565 AU's, then the chance of gravitationally bent light rays interacting solely with the plasma limb of a lensing star **would highly unlikely** produce images of Einstein rings at the site of observers beyond the plasma focal length of the lens. At astronomical distances, such light rays would be deflected completely away from all observers who are positioned beyond the plasma focal length of sun-like stars and would not be detected by these observers at all.

Because of the vast astronomical distances between the stars in our night skies, any gravitationally bent light ray would require much larger **impact parameters**, corresponding to distances clearly above the plasma limb of the lensing stars into empty vacuum space where there is virtually **no** chance for gravitational lensing effects directly caused by the stellar plasma limbs. If the light bending rule of Generally Relativity applied to the empty plasma-free vacuum space as well as the stellar plasma limbs, our night skies would be completely filled with images of Einstein rings and arcs. Thus, as a consequence of the extremely large impact parameters ξ, the bulk of the rays of starlight must propagate in the empty plasma-free vacuum space void of *indirectly interacting media* at the stellar plasma of the stars, as is confirmed by the observational evidence, i.e., **no** observations of Einstein Rings in the star-filled skies. This observation is consistent with the lack of observation of gravitational deflection of microwaves above the solar plasma limb, i.e., at the impact parameters $\xi > R$.

3 The Important Fundamentals Correctly Applied

3.1 The Fundamentals applied to the Thin Plasma Rim of the Sun

Historically, the effect of light bending has been noted only at the solar limb. The thin plasma of the sun's atmosphere appears to be due to an *indirect interaction* between the rays of starlight and the gravitational field of the sun. Figure 4 illustrates the theoretical light bending effect of the sun at various radii of analytical Gaussian surfaces, concentric to the center of the sun as suggested by General Relativity. We note in Figure 4 that the bulk of the solar lensing effect is essentially a $1/R$ effect which is predominant along a segment of the light path that encloses the

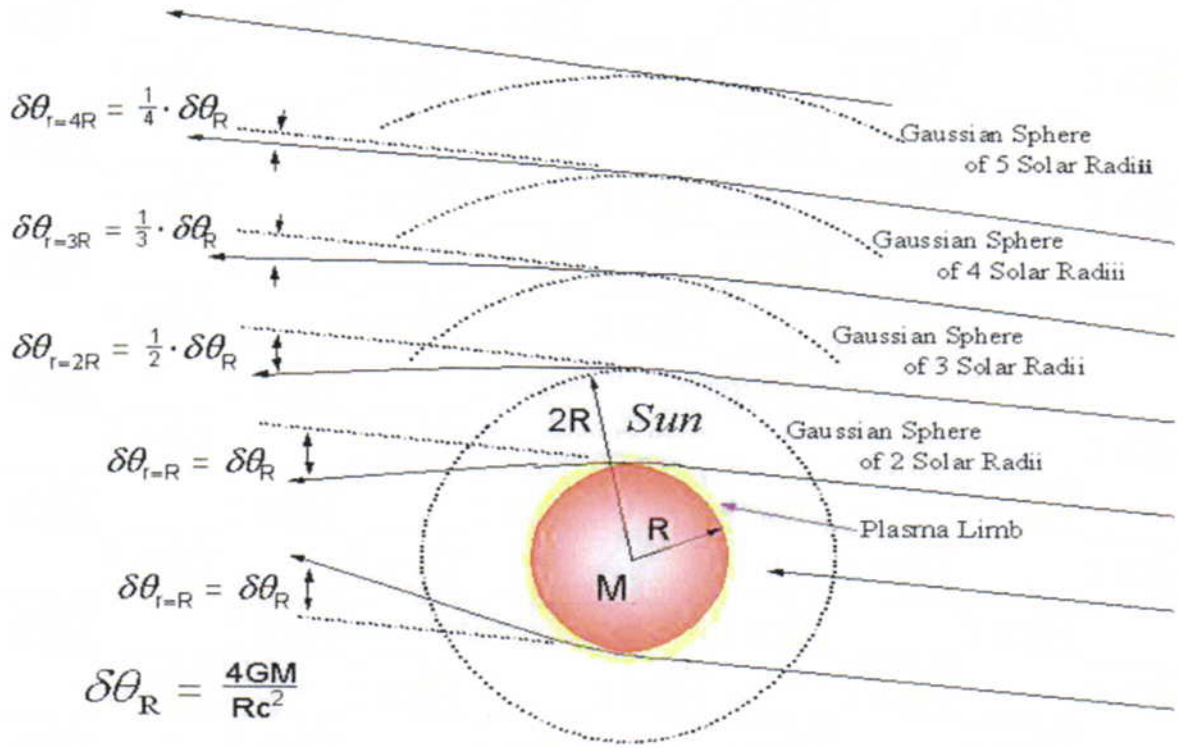

Fig. 4 Gravitational Light Bending as function of various Gaussian Surface radii of Impact Parameters $\xi = R, 2R, 3R, 4R, ...$, etc., as predicted by the light bending rule of General Relativity

impact parameter, $\xi = R$, where the indicated light ray is tangent to the spherical Gaussian surface of radius R. Since the astronomical distances are extremely large, for all practical purposes, the total integrated $1/R$ effect of the light bending occurs along a segment of the light ray that is extremely short compared to astronomical distances. Theoretically, the light bending along the segment due to the gravity of a point-like mass on approach and on receding are virtually equal and divided at the impact parameter $\xi = R$ where the relation $\delta\theta_1|_{\substack{approaching\\ the\ lens}} = \delta\theta_2|_{\substack{receding\\ the\ lens}}$ still holds even though the Earth based observer is relatively close to the sun. (Dowdye[4] , [6]) Remarkably, as it may seem, however, historically the solar light bending effect has been observed primarily at the solar limb, namely, the solar plasma atmosphere through which the light rays are directed along a *minimum energy* or *least time* path. *Dowdye*[6] The thickness of the plasma atmosphere of the sun, frequently referred to as the solar rim or the solar limb, is very negligible in comparison to the solar radius R. Table II summarizes nearly a century of observations of gravitational light bending as a function of the distance above the solar plasma limb. The bulk of the observed solar light bending events were recorded during solar eclipses. The moon has provided a near perfect masking of the solar disk, allowing only the thin plasma limb of the sun to be exposed for the astrophysical observations.

Assuming the validity of the light bending rule of General Relative, the current technical means of the astronomical

Table 2 The Observed and Predicted Gravitational Lensing at Distances h (in units of R_{SUN}) above the Solar Plasma Rim

Distance h above Rim (R_{SUN})	Gravity $1/r^2$ *Effect* (g_{SUN})	Observed Lensing (*arcsec*)	Predicted by Relativity (*arcsec*)
1.0	0.25	none	0.88
0.5	0.44	none	1.17
0.2	0.69	none	1.45
0.1	0.83	negligible	1.59
0	1.00	1.752	1.752

techniques should have easily allowed observations of solar light bending of stellar light rays at different solar radii of analytical Gaussian surfaces, namely at the radius of $2R$, $3R$ and even beyond $4R$, where R is one solar radius, as illustrated in Figure 4. For instance, at the analytical Gaussian surface of radius 2R, the predicted light bending effect of General Relativity would have been an easily detectable effect of one half the effect of 1.752 arcsec noted at the solar limb; at the surface of radius 3R, an effect of one third the effect at the solar limb, etc., etc. The equatorial radius R_{SUN} is approximately 695,000 km. The thickness of the solar limb has been recorded to be less than 20,000 km; less than 3 percent of the solar radius R. From this, we can easily see that a gravitational lensing effect in vacuum space several solar radii above the solar plasm rim should be a very noticeable effect to the modern astronomical means.

Schmeidler, et al.[17], showed that for optical wavelengths a modified corrective term that varies directly proportional to $\frac{1}{r^2}$ had to be added to the theoretical light bending effect predicted by General Relativity. The results of several observations suggested the empirical formula for the deflection of light near the sun as: $\delta r = \frac{1''.75}{r} + \frac{0''.3}{r^2}$. *Schmeidler, et al.*[17] convincingly show that the solar plasma atmosphere has very different gravitational bending effect on the rays of optical wavelengths and the rays of much longer wavelengths of microwaves. Also, at the time of the publication of their findings in 1985 the solar bending effects at or near the solar limb was not very well understood.

3.2 The Fundamentals applied to the Orbit of S2 about Sagittarius A*

The past decades of intense observations using modern astronomical techniques in Astrophysics alone reveal an obvious lack of evidence for lensing effects on collected emissions from the stars orbiting about Sagittarius A*, believed to be a super massive black hole located at the galactic center of our Milky Way. This is most obviously revealed in the time resolved images collected since 1992 on the rapidly moving stars orbiting about Sagittarius A*. (Genzel et al.[8]), (Melia[11], [12]), (Narayan[13]), (Schödel et al.[16]) The space in the immediate vicinity of a black hole is by definition an extremely good vacuum. The evidence for this is clearly seen in the highly elliptical orbital paths of the stars orbiting about the Sagittarius A*. The presence of material media near the galactic core mass would conceivably perturb the motion of the stellar object s16 which has been observed to move with a good fraction of the velocity of light. The presence of any media other than a good vacuum would have caused the fast moving stellar object s16 to rapidly disintegrate. Astrophysical observations reveal that s16 has a velocity approaching 3 percent of the velocity of light when passing to within a periastron distance corresponding to 60 astronomical units from Sagittarius A*, perceived to be a massive black hole. This gives solid evidence that the space in this region has to be, without a doubt, an extremely good vacuum. Any gravitating matter in this space would be consumed and completely gobbled up due to the intense gravitational field of the black hole. The collected emissions from the orbiting stars are in the form of ultra violent electromagnetic radiation, which are all theoretically subjected to the very same light bending rule of General Relativity. The very same rule is applied to the rapidly moving star S2 orbiting about the super massive object of approximately 4 million solar masses at the site of Sagittarius A* *as Dowdye*[6] *has shown.* It is argued whether the star S2 should appear to have entirely different orbital configuration other than that of the currently observed elliptical path. A theoretical fit to the observed orbit of S2 orbiting about Sagittarius *A and the predicted lensing of the images thereof, based on the predictions of General Relativity, was compared in this Reference. *Dowdye*[4] Some selected positions of S2 along its orbital path and the corresponding predicted lensing of the images of those positions along the orbit of S2, based on the light bending rule of General Relativity, were tabulated in the Reference *Dowdye*[4]. The magnitude of the predicted lensing effect, as would be predicted by General Relativity, should be a very noticeable effect using current technical means. To date, clear evidence of a gravitational lensing effect based on the light bending rules of General Relativity is yet to be revealed in the time resolved images of the stellar objects orbiting about Sagittarius A*. The unlikely presence of gravitating matter in the form of a light interacting media or an *indirectly interacting* light bending media at the vicinity of a black hole appears to be confirmed by the lack of evidence for gravitational lensing, as is revealed in the images of the stellar objects orbiting about Sagittarius A*.

4 Discussion & Conclusions

Historically, the light bending effect has been observed primarily at the thin plasma limb of the sun. A detail calculation obtains the very same light bending equation (1) as that obtained by the light bending rule of General Relativity, (Dowdye[6]) only this time equation (1) applies directly to the bending of light rays in a plasma atmosphere exposed to the gravitational gradient field of the sun. The calculated results of this research is confirmed by *Lebach et al.*[10] who used VLBI techniques on extra galactic radio sources to determine the gravitational deflection of microwaves at the solar plasma limb. Findings convincingly show that a *direct interaction* between the sun's gravity and the rays of starlight in the empty vacuum space at distances significantly above the solar plasma limb is yet to be observed. The celestial skies present vast opportunities to modern Astronomy and Astrophysics to allow for the detection of gravitational lensing effects, as predicted by General Relativity, due to the large numbers of stellar objects that just happen to be co-linearly aligned with the earth based observers. This, of course, assumes that the light bending rule of General Relativity applies to the plasma-free space as well as to the plasma atmosphere of the sun and the stars. Because of the vast astronomical distances between the stars, the gravitational lensing effect would have to take place in deep space, at impact parameters such that the light rays pass clearly above the plasma limb of the lensing star. If this were indeed the case and the light bending rule of General Relativity applied to a *direct interaction* between the gravitational field of the stars and the more distant rays of light in deep space, then the entire celestial sky would be filled with images of the Einstein ring. With application of the important fundamentals, the observations reveal an *indirect interaction*, **not** a *direct interaction* between the gravitational field of the lensing stars and the rays of starlight propagating in the plasma limbs of the stars. The very same fundamentals apply directly to all celestial skies and to the events taking place at Sagittarius A*. The evidence is clearly in the everyday cosmological appearance.

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A Bending of Light Rays in the Solar Plasma Rim as function of Gravitational Potential; a Minimum Energy Path Calculation

A calculation for the bending of light rays in the thin plasma limb of the sun is carried out in detail by *Dowdye*[5] and *Dowdye*[6]. The calculation is based entirely on a conservation of energy concept considering the gradient of the gravitational field of the sun acting directly on the rapidly moving ionized material particles of the thin plasma atmosphere of the sun. The calculation considers a minimum energy path for rays of light. The results is found to be totally independent of frequency. The rapidly moving ionized particles of the solar plasma is assumed to be bounded by the gravitational potential of the sun given by

$$\phi(^{r=\infty}_{r=R}) = \int_{r=R}^{r=\infty} \frac{GM}{r^2} dr = \frac{GM}{R}. \qquad (9)$$

It may be assumed that the plasma particles of the ionized solar limb move with random velocities such that their kinetic energies are as dictated by $\frac{1}{2}mv^2 = \frac{3}{2}kT + \phi m$, where m is the mean mass of the plasma particles of temperature $T(K^\circ)$ and v is the velocity of the plasma particle bounded by the gravitational potential ϕ. The velocity v of the moving ions may be assigned an upper bound of $v = \sqrt{\frac{2GM}{R}}$, the escape velocity of the solar gravity at the surface of the sun. The solar plasma particles bounded by gravity in the solar limb may be considered as a dynamic lens under the intense gravitational gradient field of the sun. It is theoretically shown here, and in detail in *Dowdye* [5] and *Dowdye* [6], that a minimum energy path for light rays propagating in the solar plasma limb, subjected to the gradient of the gravitational field of the sun, yields the mathematical results of $\frac{4GM}{Rc^2}$.

It is shown that the moving ions acting as secondary sources within the plasma limb, moving with velocities not to accede the velocity $v = \sqrt{\frac{2GM}{R}}$, the frequency and wavelength of a light ray exposed to the plasma are:

$$\nu' = \nu_0(1 - \frac{v^2}{c^2}) = \nu_0(1 - \frac{2GM}{Rc^2}) \qquad (10)$$

$$\lambda' = \lambda_0(1 - \frac{v^2}{c^2})^{-1} = \lambda_0(1 - \frac{2GM}{Rc^2})^{-1} \qquad (11)$$

$$\lambda' \approx \lambda_0(1 + \frac{2GM}{Rc^2}). \qquad (12)$$

From this, the number of wavelengths along a minimum energy path for the light ray propagating within the plasma limb may be given as

$$n = \frac{1}{\lambda'} = \frac{1}{\lambda_0(1 - \frac{2GM}{Rc^2})^{-1}} = \frac{1}{\lambda_0}(1 - \frac{2GM}{Rc^2}). \quad (13)$$

Thus, the energy ϵ per unit length of the light ray along the minimum energy path is $\epsilon = \epsilon_0(1 - \frac{2GM}{rc^2})$. Consequently, the number of re-emitted waves per unit length along the photon path and thus the energy per unit length increases as r increases. This translates to a downward, re-emitted path of the bent light ray, along a minimum energy path for the approaching segment of the light ray. If $\frac{d\epsilon}{dr} = +\epsilon_0 \frac{2GM}{r^2c^2}$ or $\delta\epsilon = +\epsilon_0 \frac{2GM}{r^2c^2}\delta R$, then the re-emission of the light ray in the atmosphere of ions will occur such that the total energy along the minimum energy (conservation of energy) path for a given light ray would not change. If ϵ is the energy per unit length along the light ray and $\delta\epsilon$ is the change in energy in the direction of the gradient potential $\phi(r)$, then the angle of change during the approach segment of the light ray is

$$\delta\theta_{app} = \frac{\delta\epsilon_{app}}{\epsilon} = +\int_{r=\infty}^{r=R} \frac{2GM}{r^2c^2}dr = -\frac{2GM}{Rc^2} \quad (14)$$

and the path change for the receding segment of the light ray is

$$\delta\theta_{rec} = \frac{\delta\epsilon_{rec}}{\epsilon} = +\int_{r=R}^{r=\infty} \frac{2GM}{r^2c^2}dr = +\frac{2GM}{Rc^2}. \quad (15)$$

The net change in the path of the light ray is

$$\delta\theta = \delta\theta_{rec} - \delta\theta_{app} = \frac{4GM}{Rc^2}. \quad (16)$$

B The Einstein Ring Equation; the General Case ($D_L \neq D_{SL}$)

The general case for the Einstein ring equation involves all values for the distances, whereby D_L is the distance between the observer and the lens and D_{SL} is the distance between the lens and the source. These are cases where D_L is not necessarily equal to D_{SL}. The general case for the radius of the Einstein ring in units of radians is

$$\delta\theta(rad) = \sqrt{\frac{D_{SL}}{D_L + D_{SL}} \frac{4GM}{D_L c^2}} \quad (17)$$

The radius of the Einstein ring at the image location the distance of $(D_L + D_{SL})$ expressed in meters is

$$R(meters) = (D_L + D_{SL})\delta\theta(rad) \quad (18)$$

where D_L and D_{SL} are also expressed in meters. The impact parameter (ξ) corresponding to the image of the Einstein ring is the nearest point of approach of the light rays to the point-like lensing mass, when observed at a distance of D_L meters away from the observer, for the rays of light coming from the light source to the observer. Since this is a 3 dimensional problem, the impact parameter of the light rays that would produce an image of an Einstein ring is in itself a ring (two dimensions). The impact parameter is a *virtual ring* for purpose of the analysis of the problem. This is illustrated in Figure 3. The impact parameter ξ(meters) is

$$\xi = R(meters) = (D_L)\delta\theta(rad) \quad (19)$$

where $\xi = R(meters)$ is the nearest point of approach of the gravitationally lensed light rays passing over the lensing star. It is that distance the lensed light rays will pass over the plasma limb of the lensing star, moving through the empty vacuum space well above the plasma limb of the lensing stars, moving along astronomical distances from the source to the observer. The radius of the predicted Einstein ring, according Equation (17) and the light bending rule of General Relativity, will be nearly 15 times the radius of a sun-like lensing star, the same mass and radius as that of the sun, when both are observed at the distance $D_L = D_{SL} = 4$ light years away, where D_L is the distance between us, the observer, and the lensing star. Adjusting the parameter D_{SL} would cause the radius of the Einstein ring to change. The means astronomical distances between the stars in our space dictates **impact parameters**, in the order of light-years, for potentially bent light rays, assuming the validity of the light bending rule of General Relativity. Increasing the parameter D_{SL} proportionally increases the image of the Einstein ring's apparent radius (an increase in magnification), again assuming the validity *direct interaction* between gravity and starlight of General Relativity. Setting $D_L = D_{SL}$, Equation (17), the general case, becomes Equation (5), the special case.